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Brief paper

A clipped-optimal control algorithm for semi-active vehicle suspensions: Theory and experimental evaluation[☆]

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ABSTRACT

This paper addresses the problem of optimal control for semi-active vehicle suspensions. A specific goal is to develop an algorithm which is capable of optimising ride and handling behaviour simultaneously in an experimental situation. A time-domain optimal control approach is adopted in which ride and handling are modelled as exogenous disturbances acting on the vehicle: road disturbances (modelled stochastically), and driver inputs (treated as deterministic quasi-static disturbances). A control algorithm is derived from a solution of the stochastic Hamilton–Jacobi–Bellman equation for the finite horizon case. The advantages of the approach are demonstrated experimentally on a test vehicle performing a steering manoeuvre on a bumpy roundabout.

1. Introduction

This paper is concerned with the design and experimental implementation of a clipped-optimal Linear Quadratic (LQ) semi-active suspension system. We focus on a suspension design framework which aims to insulate the body simultaneously from both road irregularities and handling disturbances (driver inputs, e.g. due to cornering, braking, etc.). Recent work on LQ semi-active suspension design – see for example Butsuen and Hedrick (1989), Du, Sze, and Lam (2005), Gordon (1995), Hrovat (1997), Savaresi, Poussot-Vassal, Spelta, Sename, and Dugard (2010), Sharp and Peng (2011) and Tseng and Hedrick (1994) and references therein – has often concentrated on the vehicle's response to road disturbances only. The incorporation of load disturbances into an LQ optimal control and estimation framework was proposed in Brezas and Smith (2013) in the context of active vehicle suspensions to deal with handling inputs. There, it was demonstrated, in simulation examples, that the use of a quasi-static model of the load forces is necessary both for effective control and to ensure good performance of the estimator. In the present work this approach is extended to the case of semi-active suspensions. We approach the optimal control problem by solving a stochastic Hamilton–Jacobi–Bellman (HJB) equation on a finite horizon, which motivates a constant gain clipped-optimal control law. This paper presents an experimental implementation of the algorithm on a prototype vehicle (made available as a test platform for this research) which clearly demonstrates the advantages of the approach (i.e., incorporating a model of the load disturbances in the control and estimator design). The vehicle was subjected to a slalom-type manoeuvre involving large steering inputs and significant road undulations. We provide a comparison with the standard LQG control in the literature (i.e., ignoring the load disturbance modelling), as well as a comparison with two fixed damping policies.

2. Quarter-car model and problem formulation

A typical semi-active suspension has a fixed spring k_s in parallel with a rapidly adjustable damper with damping coefficient $u(t)$ that satisfies an inequality of the form

$$0 < c_{\min} \leq u(t) \leq c_{\max}. \tag{1}$$

As usual for the control design, we take the suspension spring to be linear and we approximate the tyre by a linear spring. The equations of motion are given by

$$m_s\ddot{z}_s = F_s - k_s(z_s - z_u) - u(\dot{z}_s - \dot{z}_u) \tag{2a}$$
$$m_u\ddot{z}_u = k_s(z_s - z_u) + u(\dot{z}_s - \dot{z}_u) - k_t(z_u - z_r), \tag{2b}$$

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where z_s, z_u, z_r are the displacements of the sprung mass, unsprung mass and road, m_s, m_u, k_s, k_t are the respective mass and spring constants. In this section we assume that z_r is a Wiener process, i.e. $\dot{z}_r$ is Gaussian white noise. (For the full-car vehicle model we take a more realistic coloured noise road disturbance excitation.) As in Brezas and Smith (2013), Smith (1995) and Smith and Wang (2002) we include a load disturbance F_s on the sprung mass to approximately model the effect of handling inputs. More precisely, F_s effectively models the inertial forces induced by handling manoeuvres (such as cornering, braking, etc.) on the body, changes to static loads, as well as the aerodynamical loads and is treated deterministically. We can write the bilinear model in state-space form as

$$\dot{x} = Ax + BN^T xu + Fd + Gw, \tag{3}$$

where $x = [z_s - z_r, \dot{z}_s, z_u - z_r, \dot{z}_u]^T \in \mathbb{R}^4$, $u \in U \triangleq [c_{\min}, c_{\max}]$, $d = F_s \in \mathbb{R}$, and $w = \dot{z}_r \in \mathbb{R}$. Displacements are chosen relative to the road (rather than as absolute displacements) and the corresponding state-space matrices can be found in Brezas and Smith (2013). This has certain advantages in case the model is subjected to ramp inputs in z_r.

We make the common assumption that the adjustable damper can deliver the requested (admissible) damping instantaneously. Practical limitations would apply but these depend on the type of adjustable damper used (see Poussot-Vassal, Spelta, Sename, Savaresi, & Dugard, 2012 for more details). The reader is referred to Elmadany, Abduljabbar, and Foda (2003), Fialho and Balas (2002), Hac (1994), Hrovat (1990), Ray (1992), Ulsoy, Hrovat, and Tseng (1994), Williams and Haddad (1997), Wilson, Sharp, and Hassan (1986) and Youn, Im, and Tomizuka (2006) for further background on LQ active suspensions.

We consider the performance index

$$J = \mathbb{E}\left[\frac{1}{2}\int_0^T \left(q_0 \ddot{z}_s^2 + q_1 (z_s - z_r)^2 + q_2 \dot{z}_s^2 + q_3 (z_u - z_r)^2 + q_4 \dot{z}_u^2 + ru^2\right) dt\right], \tag{4}$$

which is to be minimised over u. We take an initial condition x_0 which is a Gaussian random vector independent of w. We note that J includes the sprung mass acceleration, tyre deflection and sprung mass displacement weightings (which are directly related to the main objectives), but also the sprung and unsprung velocity and control weightings which can in general be used for more flexibility in tuning (e.g. to achieve well-damped responses). We can write the performance index as

$$J = \mathbb{E}\left[\int_0^T l(x, u)dt\right], \tag{5}$$

where

$$l(x, u) = \frac{1}{2}\begin{bmatrix} x \\ u \\ d \end{bmatrix}^T \begin{bmatrix} Q & S_1 x & S_2 \\ x^T S_1^T & x^T R_1 x & S_3 x \\ S_2^T & x^T S_3^T & R_2 \end{bmatrix} \begin{bmatrix} x \\ u \\ d \end{bmatrix},$$

$S_1 = M_1 N^T$, $\quad S_2 = M_2$,

$S_3 = M_3 N^T$, $\quad$ and $\quad R_1 = RNN^T$.

The entries of Q, M_1, M_2, M_3, R and R_2 can be found in Brezas and Smith (2013).

3. Clipped-optimal stationary control

In this section we provide a treatment of the optimal semi-active suspension control problem for the quarter-car model that also includes a deterministic load disturbance acting on the sprung mass. We first show that an optimal control exists, and subsequently we apply the sufficient conditions for optimality to obtain an optimal control.

3.1. Optimal control formulation

For $l(x, u)$ defined in Section 2 we formalise our optimal control problem as follows:

$$\boxed{\begin{array}{c} \text{Minimise } \mathbb{E}\left[\int_0^T l(x, u)dt\right] \\ \text{over measurable } u : [0, T] \to \mathbb{R} \\ \text{and loc. abs. continuous } x : [0, T] \to \mathbb{R}^4\text{, s.t.} \\ \left\{\begin{array}{c} \dot{x} = Ax + BN^T xu + Fd + Gw, \\ x(0) = \mathbb{E}[x_0], \; u(t) \in U. \end{array}\right\}. \end{array}} \tag{P}$$

In this section we assume that the full state x is available for feedback. In Section 4.3 we describe the use of a Kalman filter to estimate the state for a full-car vehicle model.

3.2. Existence of an optimal control

Lemma 1. *The problem (P) has an optimal solution.*

Proof. It is straightforward to see that the conditions for existence of solutions in Fleming and Rishel (1975, Theorem 6.3, p. 170) are satisfied by the problem (P). □

3.3. Sufficient conditions for optimality

Theorem 2. *Consider the problem (P). Assume that, for a given initial state x_0, it is possible to find a control*

$$\bar{u} = sat_{[c_{\min}, c_{\max}]}\left\{-(N^T x)^{-1} R^{-1}\left[(B^T P + M_1)x - B^T\sigma + M_3 d\right]\right\}, \tag{6}$$

and a solution to the following boundary value problem:

$$\dot{x} = \begin{cases} (A + BN^T c_{\min})x + Fd, & \bar{u} = c_{\min} \\ (A - BR^{-1}(B^T P + M_1))x \\ \quad - BR^{-1}B^T\sigma + (F - BR^{-1}M_3)d, & \bar{u} \in (c_{\min}, c_{\max}) \\ (A + BN^T c_{\max})x + Fd, & \bar{u} = c_{\max} \end{cases}$$

where $P(t)$ is a symmetric positive-definite matrix and $\sigma(t)$ a vector satisfying

$$\dot{P} = \begin{cases} \phi_1(P), & \bar{u} = c_{\min} \\ \phi_2(P), & \bar{u} \in (c_{\min}, c_{\max}) \\ \phi_3(P), & \bar{u} = c_{\max} \end{cases} \tag{7}$$

$$\dot{\sigma} = \begin{cases} \psi_1(\sigma), & \bar{u} = c_{\min} \\ \psi_2(\sigma), & \bar{u} \in (c_{\min}, c_{\max}) \\ \psi_3(\sigma), & \bar{u} = c_{\max} \end{cases} \tag{8}$$

where

$$\begin{aligned} \phi_1(P) &= -P(A + BN^T c_{\min}) - (A + BN^T c_{\min})^T P - Q \\ &\quad - 2M_1 N^T c_{\min} - Rc_{\min}^2, \\ \phi_2(P) &= -P(A - BR^{-1}M_1^T) - (A - BR^{-1}M_1^T)^T P \\ &\quad + PBR^{-1}B^T P - Q + M_1 R^{-1} M_1^T, \\ \phi_3(P) &= -P(A + BN^T c_{\max}) - (A + BN^T c_{\max})^T P - Q \\ &\quad - 2M_1 N^T c_{\max} - Rc_{\max}^2, \\ \psi_1(\sigma) &= -(A^T + NB^T c_{\min})\sigma - (PF + M_3 N^T c_{\min} + M_2)d, \\ \psi_2(\sigma) &= -\left[A^T - M_1 R^{-1} B^T - PBR^{-1}B^T\right]\sigma \\ &\quad - \left[M_2 - M_1 R^{-1} M_3 + P(F - BR^{-1}M_3)\right]d, \\ \psi_3(\sigma) &= -(A^T + NB^T c_{\max})\sigma - (PF + M_3 N^T c_{\max} + M_2)d, \end{aligned}$$

with boundary conditions $x(0) = x_0$, $P(T) = 0$, and $\sigma(T) = 0$. Then, $\bar{u}$ is an optimal control.

Proof. The stochastic Hamilton–Jacobi–Bellman (HJB) equation is given by

$$-\frac{\partial V(t,x)}{\partial t} = \min_u \left\{ l(x,u) + \left(\frac{\partial V(t,x)}{\partial x}\right)^T (Ax+Bu+Fd) + \frac{1}{2}\mathrm{Tr}\left[\frac{\partial^2 V(t,x)}{\partial x^2} G\Gamma G^T\right]\right\}, \tag{9}$$

with $V(T,x)=0$, where $V(t,x)$ is a value function (Bryson & Ho, 1975, p. 434). It is well known that solvability of the HJB equation is a sufficient condition for optimality. Following Whittle (1996) we seek a value function of the form $V(t,x) = x^T Px + \sigma^T x + \tau$. The left-hand side of the HJB equation above is quadratic in the scalar u which has an unconstrained minimum given by

$$\begin{aligned} u^* &= -(x^T R_1 x)^{-1}\left[x^T M_1 N^T x + x^T NB^T(Px+\sigma) + M_3 N^T xd\right] \\ &= -(N^T x)^{-1} R^{-1}\left[(B^T P + M_1)x - B^T\sigma + M_3 d\right]. \end{aligned} \tag{10}$$

If $u^* \in U$ then the optimal control $\bar{u}$ equals u^*, otherwise it equals $c_{\min}$ or $c_{\max}$ and therefore $\bar{u}$ is given by (6). Given the optimal control $\bar{u}$, the HJB can be written as

$$-(x^T\dot{P}x + \dot{\sigma}^T x + \dot{\tau}) = l(x,\bar{u}) + (x^T P + \sigma^T)(Ax + B\bar{u} + Fd) + \frac{1}{2}\mathrm{Tr}\left[PG\Gamma G^T\right]. \tag{11}$$

We can have the following three cases:

(i) $\bar{u} = u^* \in (c_{\min}, c_{\max})$. Substituting $\bar{u}$ from (10) in (11) we obtain

$$\begin{aligned} & x^T\left\{\dot{P} + P(A - BR^{-1}M_1^T) + (A - BR^{-1}M_1^T)^T P \right.\\ &\quad \left. - PBR^{-1}B^T P + Q - M_1 R^{-1} M_1^T\right\} x \\ &\quad + x^T\left\{\dot{\sigma} + \left[A^T - M_1 R^{-1} B^T - PBR^{-1}B^T\right]\sigma \right.\\ &\quad \left. + \left[M_2 - M_1 R^{-1} M_3 + P(F - BR^{-1}M_3)\right] d\right\} \\ &\quad + \dot{\tau} - \frac{1}{2}\sigma^T BR_1^{-1}B^T\sigma + \sigma^T(F - BR_1^{-1}M_3)d \\ &\quad + \frac{1}{2}d^T(R_2 - M_3 R_1^{-1} M_3)d + \frac{1}{2}\mathrm{Tr}[PG\Gamma G^T] = 0. \end{aligned}$$

(ii) $\bar{u} = \mathrm{sat}_{[c_{\min}, c_{\max}]}[u^*] = c_{\min}$. From (11) we obtain

$$\begin{aligned} & x^T\left\{\dot{P} + P(A + BN^T c_{\min}) + (A + BN^T c_{\min})^T P + Q \right.\\ &\quad \left. + 2M_1 N^T c_{\min} + Rc_{\min}^2\right\} x + x^T\left\{\dot{\sigma} + (A^T + NB^T c_{\min})\sigma \right.\\ &\quad \left. + (PF + M_3 N^T c_{\min} + M_2)d\right\} \\ &\quad + \dot{\tau} - \frac{1}{2}\sigma^T BR_1^{-1}B^T\sigma + \sigma^T(F - BR_1^{-1}M_3)d \\ &\quad + \frac{1}{2}d^T(R_2 - M_3 R_1^{-1} M_3)d + \frac{1}{2}\mathrm{Tr}[PG\Gamma G^T] = 0. \end{aligned}$$

(iii) $\bar{u} = \mathrm{sat}_{[c_{\min}, c_{\max}]}[u^*] = c_{\max}$. As in the previous equation but with $c_{\max}$ in place of $c_{\min}$.

A sufficient condition for the above equations to hold for all $x(t)$ is that (7), (8) and

$$\begin{aligned} & \dot{\tau} - \frac{1}{2}\sigma^T BR_1^{-1}B^T\sigma + \sigma^T(F - BR_1^{-1}M_3)d \\ &\quad + \frac{1}{2}d^T(R_2 - M_3 R_1^{-1} M_3)d + \frac{1}{2}\mathrm{Tr}[PG\Gamma G^T] = 0 \end{aligned}$$

hold for all $t \in [0, T]$. Note that the latter equation is not needed in the theorem statement, since the optimal control does not depend on τ. From the terminal constraint it follows that $P(T) = 0, \sigma(T) = 0, \tau(T) = 0$. □

Since we are mainly concerned with the infinite horizon case we do not formally establish the existence of solutions of the system of differential equations in Theorem 2. However, we note that the system can be integrated forwards in time in a well-behaved manner with initial conditions specified on x, P, σ. It is therefore expected that shooting methods could be deployed to obtain numerical solutions of the two-point boundary value problem (Keller, 1976).

3.4. Linear Quadratic clipped-optimal control

Considering the infinite horizon, we remark that the control law of Theorem 2 does not tend to a well-defined steady-state as $T \to \infty$. Nevertheless, the form of the control law in Theorem 2 motivates the "clipped-optimal" control law stated below. A control law of similar form has been suggested in the literature (Butsuen & Hedrick, 1989; Karnopp, 1983; Margolis, 1983; Tseng & Hedrick, 1994) for the case where the load disturbance d is ignored. As in previous work we do not formally establish any optimality properties of this control law.

CLIPPED-OPTIMAL CONTROL. *Consider the problem (P) on an infinite horizon. We define the "clipped-optimal" control law by*

$$u = \mathrm{sat}_{[c_{\min}, c_{\max}]}\left\{(N^T x)^{-1} u^*\right\}, \tag{12}$$

where

$$u^* = -R^{-1}\left[(B^T P + M_1)x - B^T\sigma + M_3 d\right], \tag{13}$$

where a symmetric positive-definite P and σ are the unique solutions of the following algebraic equations:

$$\begin{aligned} & P(A - BR^{-1}M_1^T) + (A - BR^{-1}M_1^T)^T P \\ &\quad - PBR^{-1}B^T P + Q - M_1 R^{-1} M_1^T = 0, \end{aligned} \tag{14}$$

$$\begin{aligned} & \left[A^T - M_1 R^{-1} B^T - PBR^{-1}B^T\right]\sigma \\ &\quad + \left[M_2 - M_1 R^{-1} M_3 + P(F - BR^{-1}M_3)\right] d = 0. \end{aligned} \tag{15}$$

We note that under the usual assumptions that $(A - BR^{-1}M_1^T, B)$ is controllable, $Q - M_1 R_1 M_1^T \geq 0, R_1 > 0$ and $(A - BR_1^{-1}M_1^T, \hat{Q})$ (where $\hat{Q}\hat{Q}^T = Q - M_1 R_1^{-1} M_1^T$) is observable, existence and uniqueness of (14) and (15) are guaranteed.

Note that the above equations are still valid for half-car and full-car models with the division of the $N^T x$ term in (12) occurring elementwise (Hadamard product).

4. Vehicle modelling and controller structure

4.1. Vehicle model

The full-car model and road disturbance model used for the control design can be found in Brezas and Smith (2013), with the only difference that u will be element-wise multiplied by a term $N^T x$ (cf. (3)), where

$$N = \begin{bmatrix} 0 & 1 & 0 & -l_f & 0 & t_f & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -l_f & 0 & -t_f & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_r & 0 & t_r & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & l_r & 0 & -t_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T.$$

Due to left–right symmetry, the full-car model was further decomposed into two half car models (namely, bounce/pitch and roll/warp half-car models) as in Smith and Wang (2002) by a symmetric transformation

$$L_f = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix},$$

such that,

$$\left[(x)_{b_f},\ (x)_{b_r},\ (x)_{\rho_f},\ (x)_{\rho_r}\right]^T = L_f\left[x_1,\ x_2,\ x_3,\ x_4\right]^T,$$

where x can represent any of the variables z_u, z_r, u, whereas the subscripts b_f, b_r denote the front and rear bounce components and ρ_f, ρ_r the front and rear roll components respectively. This allows two independent controllers to be designed which offers more flexibility in tuning.

4.2. Clipped-optimal control

The performance index for the bounce/pitch half-car model is chosen as

$$\begin{aligned} J_b = \frac{1}{2}\int_0^\infty & q_0\ddot{z}_s^2 + q_1\ddot{z}_\theta^2 + q_2\hat{z}_s^2 + q_3\dot{z}_s^2 + q_4\hat{z}_\theta^2 + q_5\dot{z}_\theta^2 \\ & + q_6(\hat{z}_u)_{b_f}^2 + q_7(\dot{z}_u)_{b_f}^2 + q_8(\hat{z}_u)_{b_r}^2 + q_9(\dot{z}_u)_{b_r}^2 \\ & + r\left((u)_{b_f}^2 + (u)_{b_r}^2\right) dt \end{aligned}$$

and for the roll/warp half-car is chosen as

$$\begin{aligned} J_\rho = \frac{1}{2}\int_0^\infty & q_0\ddot{z}_\phi^2 + q_1\hat{z}_\phi^2 + q_2\dot{z}_\phi^2 + q_3(\hat{z}_u)_{\rho_f}^2 + q_4(\dot{z}_u)_{\rho_f}^2 \\ & + q_5(\hat{z}_u)_{\rho_r}^2 + q_6(\dot{z}_u)_{\rho_r}^2 + r\left((u)_{\rho_f}^2 + (u)_{\rho_r}^2\right) dt. \end{aligned}$$

In state-space terms each of the half-car costs can be written in a general form as in (5). Note that the $\hat{z}_s$, $\hat{z}_\theta$, $\hat{z}_\phi$, $\hat{z}_s$ and $\hat{z}_u$ denote the "relative-displacement" states as defined in Brezas and Smith (2013).

The clipped-optimal control for the full-car model is then computed by

$$[u_1,\ u_2,\ u_3,\ u_4]^T = \mathrm{sat}_{[c_{\min},c_{\max}]}\left\{(N^T x)^{-1} \circ \left[u_1^*,\ u_2^*,\ u_3^*,\ u_4^*\right]^T\right\},$$

where $\circ$ denotes the Hadamard product (i.e., elementwise multiplication) and

$$\left[u_1^*,\ u_2^*,\ u_3^*,\ u_4^*\right]^T = L_f^{-1}\left[(u)_{b_f}^*,\ (u)_{b_r}^*,\ (u)_{\rho_f}^*,\ (u)_{\rho_r}^*\right]^T,$$

where $((u)_{b_f}^*, (u)_{b_r}^*)$ (resp. $((u)_{\rho_f}^*, (u)_{\rho_r}^*)$) are computed from (13) corresponding to the bounce/pitch (resp. roll/warp) half-car model.

4.3. State estimation

A straightforward approach to the state estimation is to compute the (deterministic) body load disturbances F_s, T_θ, T_ϕ directly from the equations of motion. This is feasible in typical cases, e.g. if suspension deflection and body acceleration measurements are available. The (stochastic) Gaussian input to the road disturbance low-pass filter is treated as the input noise with covariance matrix Γ in the Kalman filter and output noise with covariance matrix Δ is assumed on the sensor measurements, with a full-car model written in the form

$$\dot{\breve{x}} = \breve{A}\breve{x} + \breve{B}u + \breve{F}d + \breve{G}w_r,$$
$$y = C\breve{x} + Du + Hd,$$

where $\breve{x}$ denotes the vector comprised of the state vector $\hat{x}$ and the road model state as in Brezas and Smith (2013).

It can be checked, with the available measurements and the body loads calculated as above, that $(C, \breve{A})$ is detectable. Also, the pair $(\breve{A}, \breve{G}\sqrt{\Gamma})$ is stabilisable. The observer then takes the form

$$\dot{x}_e = \breve{A}x_e + \breve{B}u + \breve{F}d + K_e(y - y_e), \tag{16}$$

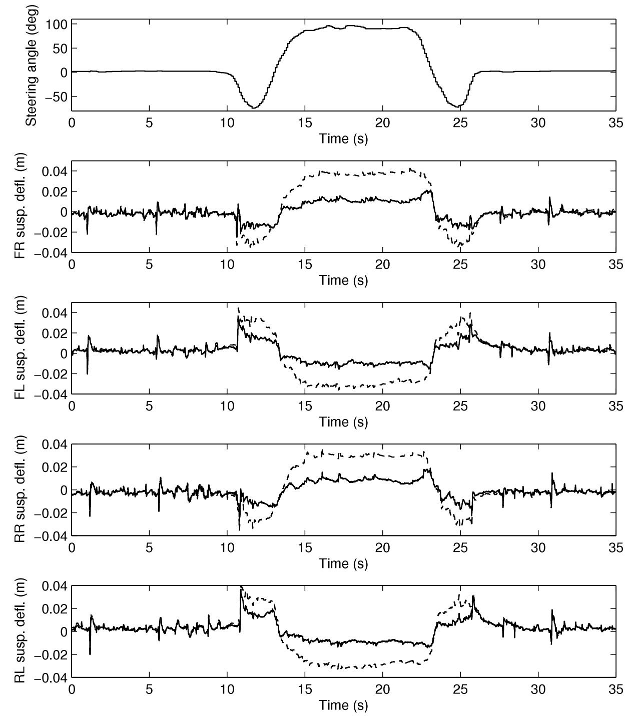

Fig. 1. Steering wheel angle, (solid) measured suspension deflections and (dashed) estimated suspension deflections for the estimator with no disturbance modelling for a roundabout manoeuvre (60 km/h).

where x_e is the estimation of the state vector,

$$y_e = Cx_e + Du + Hd, \tag{17}$$

and $K_e = P_e C^T \Delta^{-1}$ is the standard Kalman filter gain where P_e satisfies the Riccati equation

$$\breve{A}P_e + P_e\breve{A}^T - P_e C^T\Delta^{-1}CP_e + \breve{G}\Gamma\breve{G}^T = 0.$$

5. Experimental results

The algorithm developed in this paper was evaluated experimentally on a test vehicle (high-performance sports car) with suspension consisting of a continuously adjustable damper in parallel with a passive spring. The damper is of variable-orifice type and has a bandwidth of 60 Hz. In the test setup the physical damper has a nonlinear velocity–force characteristic. In the experiments reported here the bounds $c_{\min}$ and $c_{\max}$ are velocity-dependent, effectively to allow the damper to provide its maximum force in the saturation region, rather than restricting to an artificial maximum given by predetermined fixed $c_{\min}$ and $c_{\max}$. This practice was followed in all experiments for a fair comparison. The test vehicle parameters were in the typical range for a high-performance sports car. The cost weightings were chosen to adjust the trade-offs for ride comfort, tyre grip, handling etc., for achieving satisfactory performance for different driving modes. The weights on the body accelerations and tyre deflections gave good authority for achieving a good compromise between ride comfort and tyre grip during objective tests like a bumpy road profile or symmetric and asymmetric bumps. The roll angle and roll velocity weights were found to be useful tuning parameters for handling behaviour. Note that in the comparison between the proposed control scheme and the standard clipped optimal approach the performance weights are the same. The available measurements are the suspension deflections and the body and hub accelerations. The control algorithm is implemented in discrete form with a 1 ms sampling period. Experimental results are now presented for a steering manoeuvre around

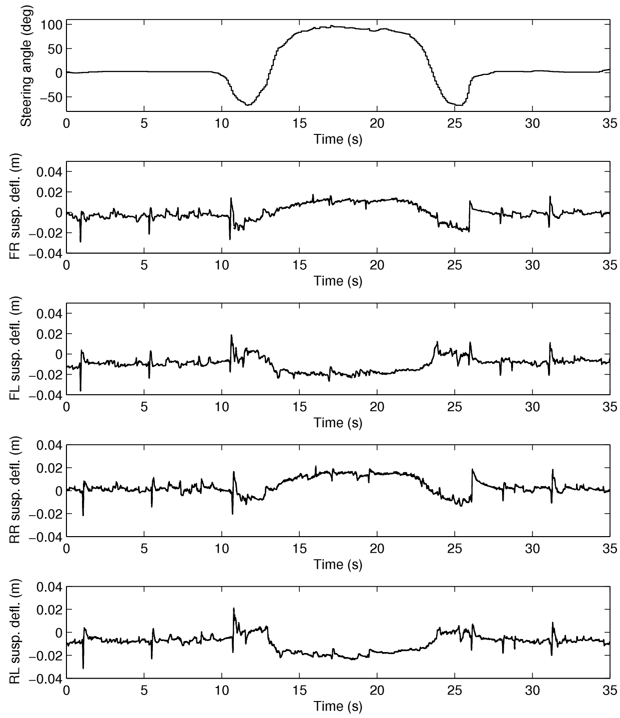

Fig. 2. Steering wheel angle, (solid) measured suspension deflections and (dashed) estimated suspension deflections for the estimator with disturbance modelling for a roundabout manoeuvre (60 km/h). Note that the measured and estimated signals coincide in the plot.

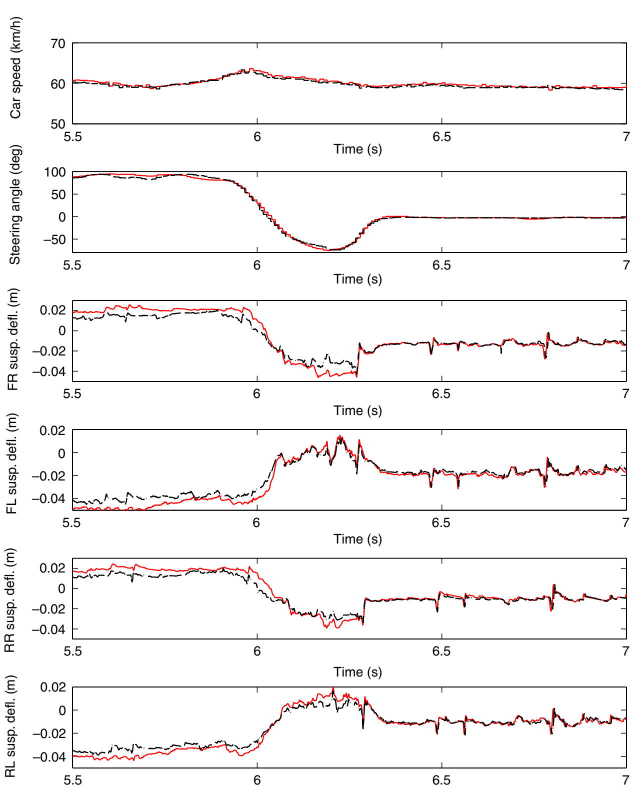

Fig. 3. Car speed, steering wheel angle and measured suspension deflections when using the observer with disturbance modelling (black-dashed) and when using the observer with no disturbance modelling (red-solid). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

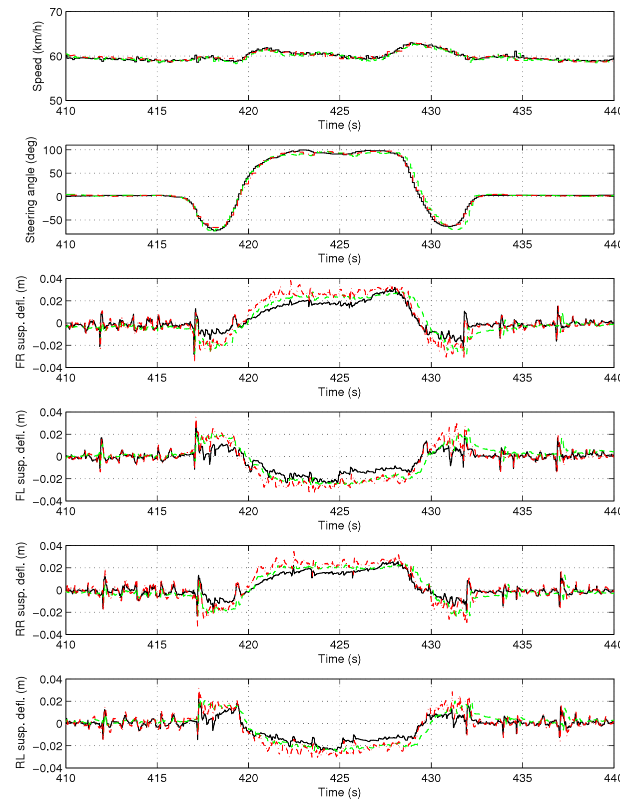

Fig. 4. Car speed, steering wheel angle and measured suspension deflections for (black) the proposed semi-active control, (green) a "stiff" fixed damping setting, and (red) a "soft" fixed damping setting. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

a bumpy roundabout at 60 km/h, which offers simultaneously significant ride and handling inputs, and repeatability.

5.1. Influence of load disturbances on estimation quality

Fig. 1 shows the estimated and measured suspension deflections of the test vehicle when the load disturbances are ignored in the estimation design (i.e., setting d to zero in (16), (17)), for the described steering manoeuvre. We observe that while the estimation is satisfactory just before and after the steering input, it fails to be so during the steering input. On the other hand, repeating the manoeuvre with the observer taking into account the load disturbances as proposed in Section 4.3, the estimation is not affected during the steering input as shown in Fig. 2. In both cases the control law used is that given in Section 4.2.

5.2. Comparison of the observer with and without disturbance modelling

As a measure of the effects in performance, a comparison of the two systems in measured suspension deflections is shown in Fig. 3. We observe that for the controller with Kalman filter with no disturbance modelling we have an average increase in the measured suspension deflections of the order of 1 cm during the steering input, whereas after the steering input the responses are almost identical. The increase in measured suspension deflections can be regarded as a significant handling quality degradation, as a 1 cm increase in suspension deflection would correspond roughly to a 1.5° increase in roll angle, given also that the speed of the car is relatively low (60 km/h). In terms of ride comfort, the rms value of the sum of the body accelerations for the observer without disturbance modelling was increased by an approximately

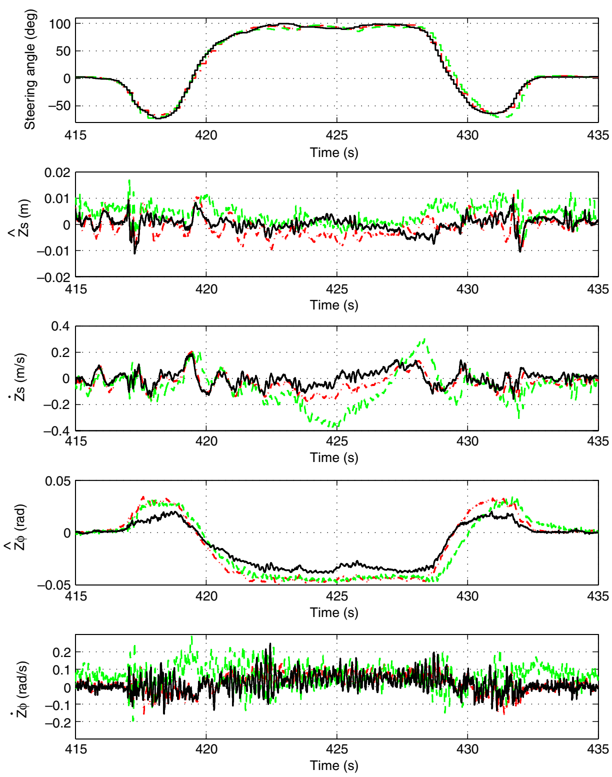

Fig. 5. Steering angle, estimated (relative) heave displacement, estimated heave velocity, estimated (relative) roll angle and estimated roll velocity for (black-solid) the proposed semi-active algorithm, (green-dashed) a "stiff" fixed damping and (red-dashed–dotted) a "soft" fixed damping setting. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

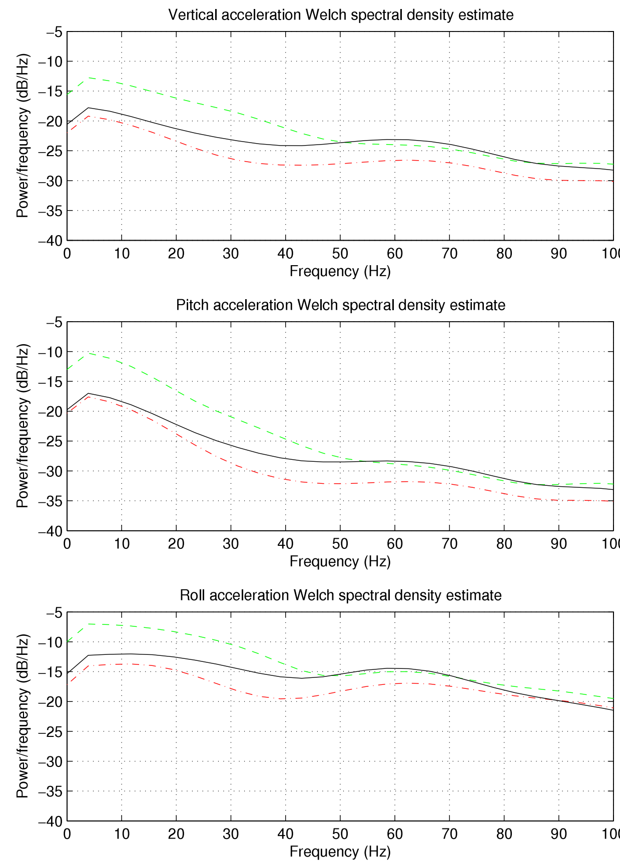

Fig. 6. Power spectral densities of the measured body accelerations for (black-solid) the proposed semi-active algorithm, (green-dashed) a "stiff" fixed damping setting, and (red-dashed–dotted) a "soft" fixed damping setting. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

5% compared to the proposed observer. We note that despite the wrong estimation while cornering, the car does not exhibit instability issues, due to the passive nature of the forces generated by the adjustable damper.

For the case where disturbance modelling is included in the estimator but not in the control law there is also a performance degradation for the given manoeuvre, but of lesser extent than in Fig. 3.

5.3. Comparison with fixed damping suspension

Fig. 4 shows the measured suspension deflections for the proposed semi-active algorithm (incorporating disturbance modelling and estimation) and for two fixed damping settings ("soft" and "stiff"). We observe that during the manoeuvre the proposed semi-active system achieves smaller suspension deflections than either of the fixed damping settings both in the transient and steady-state phases. It is interesting to note that the combination of the (nonlinear) semi-active control algorithm and the road excitation produces a smaller mean offset during the steady-state part of the manoeuvre than is possible with a spring and fixed damper. Furthermore, a comparison of the estimated roll and heave body states is shown in Fig. 5 where we observe significant improvements for the proposed semi-active algorithm, especially for the roll angle which is a key parameter for a steering manoeuvre. Pitch states are not shown as they are not of significant magnitude.

In Fig. 6 we see that the power spectral densities (PSDs) of the body acceleration measurements for the semi-active algorithm lie between those of the fixed damping settings and, in fact, they are closer to the "softer" one in the more important lower frequency range. In terms of PSDs of the hub accelerations we observe in Fig. 7 that for frequencies of up to approximately 15 Hz the semi-active algorithm is as good as the "stiff" setting and significantly better than the "soft" setting and vice versa for frequencies of around 25 Hz and onwards. For the range of frequencies 15–25 Hz (approximately) the semi-active algorithm gives lower values of PSDs than either of the fixed damping settings.

6. Conclusions

This paper was concerned with the design and experimental implementation of a clipped-optimal LQ semi-active suspension system. The design framework was generalised by incorporating the handling inputs in the control problem formulation. An experimental implementation and testing of the semi-active control algorithm on a prototype vehicle was presented. Ignoring the load disturbances in the design was shown to have a deleterious effect on the state estimation during handling manoeuvres. The proposed control method maintained satisfactory state estimation during handling manoeuvres, and achieved improved performance compared to standard LQ approaches. A practical comparison between the proposed semi-active suspension system and fixed damping settings was also presented, demonstrating the effectiveness of the semi-active control method in achieving a better performance compromise.

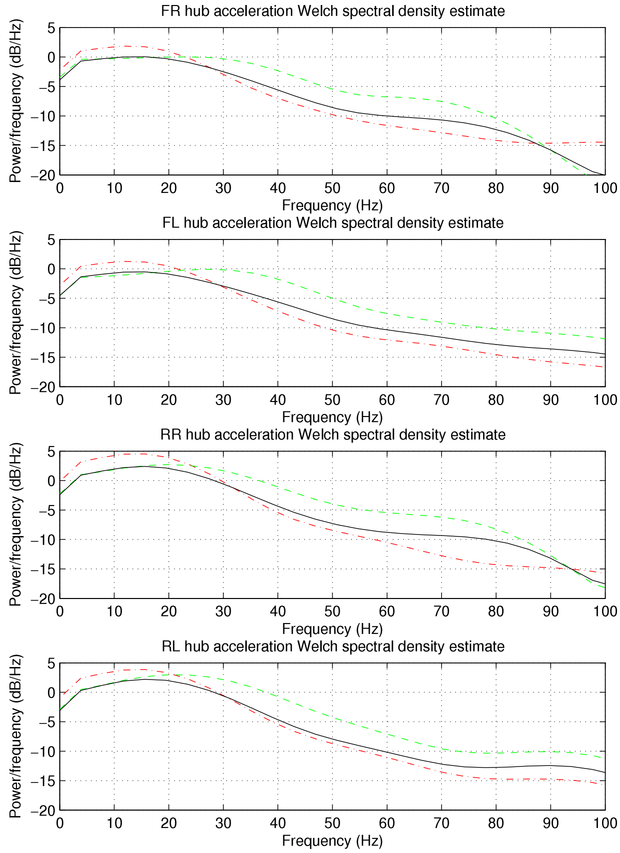

Fig. 7. Power spectral densities of the measured hub accelerations for (black-solid) the proposed semi-active algorithm, (green-dashed) a "stiff" fixed damping setting, and (red-dashed–dotted) a "soft" fixed damping setting. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

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